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About Various Methods of Calculating the Sum $\sum_{k=1}^n k^m$

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Abstract

Pupils of secondary school as well as students often have problems with calculating the sums of the m th powers of successive natural numbers. In this paper we present certain methods of finding such sums.

On Some Special Morphisms Between Groups and Algebras

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Abstract

A classical trigonometric identity states that

$$(\sin x)^2 + (\cos x)^2 = 1 \quad \text{for all } x \in R.$$

Replacing here the sine and the cosine by functions $f, g : R \rightarrow R$, and 2 by a natural number k , we receive the functional equation:

$$(I) \quad (f(x))^k + (g(x))^k = 1$$

with $x \in R$, which was studied by R. Tardiff in [3], in connection with some trigonometrical considerations. In my papers [1] and [2] I gave a partial answer to the question of the professor Roman Ger concerning addition formulas: $f(x+y)$ and $g(x+y)$ for the function f, g satisfying (I), which would correspond to the well known representations of $\cos(x+y)$ and $\sin(x+y)$ then $x, y \in R$ in the case where $k = 2$. In [1] I introduced such formulas in the case of even k , the case of odd k was considered in [2]. The results contained in the paper [1] yield the starting point for considerations of the present paper.

3. **Górnicka Anetta**
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Formalization of the Sentential Logic Dual to Łukasiewicz's Three-valued Logic

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Abstract

Dual logics with respect to Łukasiewicz's logics were investigated by G. Malinowski, M. Spasowski and R. Wójcicki in [4,5]. Our aim is to discuss the generalized method of natural deduction for the logic which is dual to Łukasiewicz's three-valued logic.

4. **Grondys Kamil, Kurkowski Mirosław, Sowik Anna, Szczypior Izabela**
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Verifying Untimed Version of The WMF Protocol Using Networks of Automata¹

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Abstract

In this paper we present some results of symbolic verification of untimed version of The Wide-Mouth Frog Protocol. This protocol is designed for achieving authentication between communicating sides in the computer network and exchange a new session cryptographic key. For our investigation we model executions of this protocol by a network of synchronized untimed automata. We investigate suitable protocols properties by testing reachability of some distinguished states in the defined network. We use VerICS [5] - the symbolic model checker - for searching in our network.

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5. Kurkowski Mirosław, Małek Jacek,
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On Some Crypto-Messages Parser 59

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On Some Crypto-Messages Parser²

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Abstract

Some methods of automatic verification of cryptographic protocols require creating specially designed formal languages based on suitable algebra of terms (called *crypto-terms* or *crypto-messages*). Sometimes in verification process it is essential to check whether a given crypto-term is a subterm of another one or in general whether some crypto-term belongs to the specified crypto-language (a set of some crypto-terms) or not. Another problem is creating the set of all antichains of subterms of a given term with respect to the order introduced by a subterm relation. The main goal of this paper is to present a tool for verification whether a crypto-term belongs to the specified crypto-language or not. Our main purpose is to check lexical analysis and syntax analysis. Finally, after checking that a crypto-sentence is correct, we convert it into the special format, which can be transformed to an atomic version of this sentence, and make an antichain. We can introduce formulas manually or as text files. The output may also be changed into a text file. The program is a text tool which is started in a Linux environment.

²This research is partially supported by the Ministry of Science and Information Society Technologies under the grant number 3 T11C 011 28.

6. **Pisarek Jerzy**
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Knot Point Method in Extrapolation of Boundary Measurement Results

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Abstract

The new measurement technique that relies on distinguishing the ranges in object's space in which the searched quantities are approximated by multiparameter continuous functions has been introduced. The parameters of these functions are calculated by minimalizing the functional describing the distance between different functions determined at the same points and distance between numerical solution and experimental results and assumed 'a priori' theoretical guidelines. Functions' spaces might be determined in any way and might be changed during the computation. This method is especially useful in extrapolation of the boundary measurement results on the object interior and in joining the local and all object (global) solutions.

7. **Povstenko Jurij**

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Two-dimensional and One-dimensional Balance Equations and Their Applications

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Abstract

An interfacial region and a three-phase line region are considered as two-dimensional and one-dimensional continua. Equations of the linear momentum balance and moment-of-momentum balance generalize the Laplace equation for surfaces and the Young equation for lines. Balance equations for surface dislocations and disclinations are also considered. The motor analysis is used for a description of continua with couple stresses.

8. **Tikhonenko Oleg, Ziółkowski Marcin**
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Queueing Systems with Random Volume Internal and External Calls

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Abstract

In the present paper we investigate a single-server $BM/G/1/\infty$ queueing system with non-homogeneous calls of two following types: 1) external calls served by the system under consideration, 2) internal calls arrive only when an external call is served and interrupts the service process. The external calls appear according to a stationary Poisson process with bulk arrivals. Calls of each from above-mentioned types are characterized by some random volume. Service time of the call arbitrarily depends on its volume. Two schemes of calls service organization are analyzed. The non-stationary and stationary total calls volume distribution is determined in terms of Laplace and Laplace-Stieltjes transforms. The stationary first moment of total calls volume distribution is calculated for each scheme.

9. **Wójcik Wiesław**
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The Ancient Origin of Symmetry Idea and the Newman Property

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Abstract

The vital significance for development of Greek mathematics lied in the idea of an inner similarity – self-reconstruction and self-duplication of a given structures. That way the idea of symmetry was born which was understood by Greeks as a conformity between parts and wholeness as well as between individual parts of wholeness. Consequently, the idea of similarity is a particular case of the idea of symmetry. Idea of similarity shows invariance of some elements (e.g. the ratio of suitable similar polygon sides) under some kind of transformations. Transformations of space in itself, which preserve its internal structure, are named automorphisms. Hence, this is symmetrical (as a part of a space) what is preserved by automorphisms of the space. Leibniz expressed this thought in philosophical way saying that such two things are symmetrical which we can not diverse when we consider them in itself.